

Assignment Problems in Wildfire Suppression: Models for Optimization of Aerial Resource Logistics

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1 Assignment problems in wildfire suppression:
2 Models for optimization of aerial resource logistics

3

4 **Abstract**

5 Wildfire containment activities involve a combination of important
6 decisions that affect the evolution of the fire and effective resource
7 deployment. When aerial resources (in particular aircraft and heli-
8 copters) are used, two tasks are assigned to the aerial coordinator: the
9 allocation of aerial resources to flight routes (circular paths that aerial
10 resources follow such that they have common loading and discharge
11 points) and refueling points.

12 In this paper, we introduce two models of linear integer program-
13 ming to execute these tasks. The models are written using AMPL and
14 the Gurobi solver engine and illustrated through examples.

15 The objective of these models is to provide automatic and rapid
16 support for the coordination of the above mentioned tasks. In order
17 to enhance the robustness of the models, the scheduling times and
18 the characteristics of the aerial resources are also considered. These
19 models aim at minimizing both the containment time of the fire and
20 the total flight hours. The models will reduce the risk of aerial col-
21 lision of resources by taking into account the maximum number of

22 aerial resources that can simultaneously load water at the same point.

23 Moreover, management of refueling points is also achieved.

24 **Keywords:** *wildfire management, aerial resources assignment, flight*
25 *routes, refueling points, integer linear programming.*

26 **Introduction**

27 There has been extensive research on the propagation and containment of
28 wildfires. It must be kept in mind that the evolution of fires, the ecologi-
29 cal impact thereof, the socioeconomic impact, and underlying problems in
30 fire management decisions are interrelated issues. This article addresses the
31 fourth point, and we call attention to the fact that in recent decades, there
32 have been numerous studies on fire management decision making (Miller and
33 Ager, 2013). In terms of fire management, noteworthy aspects are preven-
34 tion, detection, and management of fire extinguishing resources (Minas *et*
35 *al.*, 2012).

36 In wildfires, the presence of several aerial resources working at once (some
37 of them perhaps from other administrations) with different points for loading
38 and discharging water (and other supplies such as foam fire suppressants and
39 water enhancers) lead to an increase in air traffic. This increase in the de-
40 ployment of aerial resources in wildfires brings with it a risk of air collisions.
41 For example, cases where two flight routes share the same water intake but
42 have different point of discharge or situations where aerial resources exceed
43 the maximum work time as they wait for orders. Therefore, air traffic coor-
44 dination is imperative to determine aerial resource instructions, flight routes,

45 work duration, and risk of collision.

46 The design of decision support systems for wildfire containment is an
47 active area of research in modern Operations Research, producing a num-
48 ber of applications. In this study, we focus on two tasks—the assignment
49 of aerial resources to flight routes and the assignment of aerial resources to
50 refueling points. In particular, if we suppose that the number and type of
51 aerial resources have been determined, the first problem is to decide how to
52 maximize the amount of water discharged on the fire affected areas. This is
53 achieved by the appropriate allocation of aerial resources along flight routes
54 subject to the following restrictions: no fronts are left unattended, the max-
55 imum number of aerial resources is respected by the flight route (a number
56 determined by the coordinator), and the percentage of water for each front,
57 chosen by the coordinator, is delivered accordingly.

58 Moreover, when the aerial resources that are assigned to a wildfire begin
59 their resting period, they have to refuel. Assigning these aerial resources to
60 refueling bases is a complex task because of the various factors involved, such
61 as the time of arrival at the refueling points and the amount of fuel available
62 at each point. Therefore, the second problem is to ensure optimal allocation
63 of aerial resources to the refueling points such that the total time taken by
64 all aerial resources in the operation is minimized. In the following sections,
65 we discuss the decision problem that we want to address, the literature in
66 which this problem is framed, and the methodology used for its resolution.

67 The problems

68 In Spain, compared to the relatively recent past, the number of aerial re-
69 sources that can coincide in time and space in a forest fire has increased
70 considerably. It was not until the 1970s that the aerial resources were de-
71 ployed to fight forest fires. The organization of the air strike did not pose
72 complications and was developed among the aerial resources pilots them-
73 selves. However, in the present-day scenario, lack of proper air coordination
74 substantially compromises flight safety as well as the effectiveness of the
75 mission (*cf.* Vélez, 1999).

76 Consequently, in the past decade, several research projects have been
77 carried out by companies from the public and private sector domains. The
78 main objective of these projects was to develop advanced technologies to fight
79 wildfire fires, thus reducing their number and scope, and to create a safety
80 protocol that significantly reduces the accident rate (technical, brigade, and
81 pilots).

82 In Spain, the airspace is classified by four 500-ft altitude intervals, de-
83 pending on the type of aerial resources (*cf.* Couceiro-López, 2007). Aircraft
84 aerial coordinators operate in the fourth interval (more than 1500 ft), which
85 enables them to perform their work more efficiently. The air operations
86 coordinator is responsible for the operations and the organization of air as-
87 sets in the event of a forest fire. The main objective of the air operations
88 coordination is to ensure the safety and efficacy of the air assets involved
89 in the operations. The altitude interval of 1000–1500 ft comprises the flight
90 routes of heavy aircraft, which are characterized by higher tonnage and cruis-

ing speed. Light aircraft fly at altitudes between 500–1000 ft, as they are slower and less powerful, but more economical and with greater maneuverability. Helicopters fly at lower altitudes (less than 500 ft) as they offer greater maneuverability albeit at slower cruising speeds. So, this division of airspace makes it possible for aerial resources with similar characteristics to be grouped together. Moreover, the division of airspace enables the simplification the models, reducing variability in fuel consumption by weight, speed, and altitude; these variables can be considered constant for each flight route.

Specifically, helicopters, both monoturbine and biturbine, are light and easy to maneuver (Bell 407, Eurocopter AS350 Series B3, or Eurocopter AS355N, among others) or heavy and capable of carrying large amounts of water that can be dropped anywhere except the most virulent foci (Ka32 or Kamov AS330J Puma Eurocopter). Airplanes can be classified, as in the case of helicopters, as light (Air Tractor AT802) and heavy (Canadair CL215); each type has different functions. One of the most important aspects of managing flight routes is the homogeneity of the aircraft; grouping similar aerial resources produces uniform cadence of the flight route. Aerial resources work is organized into flight routes, such that groups of aerial resources fly over the fire affected area by forming a circuit pattern, from which each aerial resource has access to a water intake point. Naturally, if these operations are disorganized (*i.e.*, the status of each aerial resource is unknown), they pose a high risk of collision.

The fuel used by the aerial resources is limited, and in large forest fires, more than one instance of refueling per aircraft is necessary on a given day. In Spain, as defined in the regulation *Circular Operativa 16-B (Dirección*

116 *General de Aviación Civil, Ministerio de Fomento de España, 1995*), refu-
117 eling is performed while the aerial resources rest on the ground. As per
118 these regulations, aerial resources must have a minimum 40-minute break
119 between every two hours of consecutive flight. Therefore, to realize efficient
120 operations, it is important that refueling does not exceed the break period.

121 **Theoretical framework**

122 The location of resources and their selection for the so-called initial attack
123 (namely, the actions taken by the first resources to arrive at a wildfire to
124 protect lives and property, and prevent further extension of the fire) have
125 been studied in literature using models from operations research as well as
126 simulation tools. The assignment problem is a classical problem in linear
127 programming, having first appeared in the work of Votaw and Orden (1952)
128 and becoming more widespread with the publication of the Hungarian solu-
129 tion (*cf.* Kuhn, 1955). A recent review of this problem and its generalizations
130 can be found in Pentico (2007).

131 Determining the optimal flight plan, including the number and type of
132 resources needed to extinguish a forest fire, is not an easy task given the wide
133 range of possibilities. In Islam and Martell (1998), a tool is proposed for de-
134 signing air tanker dispatch policies that minimize the initial attack response
135 times. They also stress the importance of taking into account the traffic
136 congestion in aerial resources, which is considered in the models presented
137 in this work. The first model we propose is similar to a three-dimensional as-
138 signment problem (*cf.* Geetha and Vartak, 1994) because the aerial resources

139 are assigned to flight routes that are in turn allocated to different fire fronts.
140 However, a slightly more general situation arises here because several ob-
141 jectives are simultaneously considered (*cf.* Geetha and Nair, 1993). Given
142 that the purpose is to maximize water discharge on the fronts and minimize
143 distances between aerial resources and flight routes, specific restrictions are
144 introduced in aerial resources and water points related to capacity as well as
145 preferences in terms of the percentage of water received at the fronts.

146 A tactical decision model, which determines the optimal combination
147 of suppression resources to minimize a certain cost function, was proposed
148 by Donovan and Rideout (2003). The models put forth in this work are
149 similar in certain respects to the abovementioned models. However, this
150 study emphasizes on the problem of maximizing water disbursement and
151 minimizing refueling times.

152 The quality of data and its availability can limit the subsequent analysis.
153 We believe that this issue is relevant, and in fact, there have been a number
154 of recent works published on this topic, including Calkin *et al.* (2014) and
155 Stonesifer *et al.* (2016). In these works, statistical studies are performed
156 wherein variables such as the number of water downloads by air assets, size
157 of fire, and time of day are considered. They mention the problem of the
158 lack of data regarding the number of downloads and their effectiveness and
159 the importance of finding a solution to this problem. This fits with our work
160 because although our algorithms offer a response to the demand for wildfire
161 coordinators, it is necessary to have sufficient data to reliably verify the effec-
162 tiveness of operations and to ascertain the need for possible improvements.

163 In Dimopoulou and Giannikos (2004), an integrated tool comprising a

164 geographic information system, a linear programming model, and simula-
165 tion tools is proposed to address the problem of allocating land resources to
166 different areas of a fire. With the addition of tools designed by our team,
167 our study can complement the integrated tool because we address additional
168 issues such as air assets, flight routes, and refueling points.

169 Martell (2015) offers a review of the existing research on recent forest and
170 wildfire management decision support systems. The author also describes a
171 general working procedure for wildfire fires, which states that “In the case
172 of amphibious air tankers, the air attack officer must decide from which
173 body of water each air tanker picks up water and when and where each air
174 tanker drops its load”. This is the starting point of our work, and hence it
175 is emphasized.

176 **Methodological approach**

177 We propose two models of linear integer programming to solve the two deci-
178 sion problems described in the introduction and framed in the line of optimal
179 allocation of fire extinction resources.

180 The first model is designed to maximize the output per hour of aerial
181 resources flight time. As regards the sets aerial resources (helicopters and
182 airplanes), we group these resources such that the resources of the same
183 group can be integrated (all of them or only a proper subset) along the same
184 flight route. We also consider a set of water points and a set of fire fronts.
185 In general, each resource can only be assigned to a certain water point.

186 With respect to the parameters of interest, the coordinator must know

187 the capacity (in liters) of each water point. The coordinator must also know
188 the maximum number of resources for each group that can be assigned to a
189 single flight route, which consists of a front and water point. In addition, the
190 following are also known: the number of downloads per hour for an aerial
191 resource, the distance from the resource location to each front, the number of
192 active flight routes that share a given water point, and the amount of water
193 (expressed as the relative frequency) as a proportion of the total capacity of
194 resources that is intended for each front.

195 It should be noted that cruising speeds and topographical features are
196 not explicitly considered. However, the performance on the flight routes
197 implicitly reflects these elements. More precisely, the yield is estimated by
198 statistical regression functions, the details of which we omit here, based on
199 the characteristics of the aerial resources and the distances between the water
200 points and fronts; this allows the coordinator to estimate the time required
201 for an aerial resource to travel the route, which is, in fact, the time between
202 two consecutive water discharges.

203 Another important aspect is the priority (threat-based ranking) assigned
204 to the different fronts of a wildfire. In Spain, air coordinators allocate more
205 water to the most important fronts. We must emphasize that this model
206 only considers those fronts that are selected by the coordinator for attack.

207 The second model manages the allocation of aerial resources to refueling
208 bases. The model should take into account the following aspects. First, it
209 should consider the number of aerial resources that can simultaneously refuel
210 at a given base. For example, if a refueling point is a tanker in the middle of
211 an open field, *i.e.*, a single tanker with a single hose, the simultaneous supply

212 of fuel to multiple aerial resources becomes impossible. The amount of fuel
213 at each base and the fuel capacity of each aerial resource are also relevant
214 factors. Another possible scenario is one where the refueling base is close to
215 the fire, but the amount of available fuel is less than that required by the
216 aerial resource. Multiple aerial resources should not be assigned to the base
217 despite its proximity to the fire. Moreover, an aerial resource may prefer to
218 wait in the air while another aerial resource completes refueling as opposed
219 to going to a base further away (thereby losing time), except in the case
220 where the time spent waiting is greater than having to go to another base.
221 Moreover, once the optimal allocation is determined, the aerial resources are
222 issued a warning regarding the new capacity of refueling points, to ensure
223 that fuel is replenished where necessary.

224 As regards the sets for which we need information, we consider a set of
225 resources (again helicopters and airplanes), a set of refueling bases, and a
226 set of periods of time. As regards the parameters, we take into account the
227 fuel load of each aerial resource, the refueling time required by each aerial
228 resource, the quantity of fuel available in each base, the number of aerial
229 resources that can refuel simultaneously on each base, and the time it takes
230 to move each aerial resource to each base. In addition, we must consider
231 that some aerial resources do not have the capability to refuel at all bases.

232 It is important to remark that this paper introduces two models to solve
233 common tasks during a wildfire suppression operation. These two tasks are
234 interrelated because they begin at the instance when an aerial resource fleet
235 is assigned a schedule. This is a common and difficult problem for wildfire
236 coordinators to solve. To illustrate the use of the proposed models, we in-

237 troduce a scheme that a coordinator should follow. However, the time that
 238 this decision takes place may be right at the beginning of the fire extinction
 239 protocol, or at different times during the extinction process of the said fire.
 240 This is because aerial resources are assigned different areas, and need time
 241 to refill their water tanks. Hence, when a set of aerial resources are assigned
 242 roles in the fire extinction process, the coordinator is faced with the prob-
 243 lem of deciding which aerial resources have to attack which front, and their
 244 corresponding refill points (*i.e.*, the aim of the allocation model of aerial re-
 245 sources to flight routes). After the allocation of the task by the coordinator,
 246 the aerial resources accordingly start working on the fire. Subsequently, a
 247 set of aerial resources presently working on the fire will be required to take
 248 breaks (due to aviation regulations). At such instances, the aerial resources
 249 will perform refueling operations. It is at this moment that a new problem
 250 arises, the problem of assigning the aerial resources to the refueling points
 251 (*i.e.*, the aim of the allocation model of aerial resources to refueling points).
 252 To address this problem, time discretization is essential because of the wait-
 253 ing time of each aerial resource needed to carry out these tasks. Once the
 254 refueling task is completed, the aerial resources will return to their previ-
 255 ously assigned work plan. Therefore, it is paramount to consider the time
 256 needed to fly to the refueling point as well as the time needed to return.

257 After contextualizing the use of these two models, it is important to em-
 258 phasize when they should be executed. The first model would be executed/re-
 259 executed whenever a new aerial resource enters or abandons the extinguishing
 260 protocol as well as when any relevant changes in the evolution of the fronts
 261 occur. The resolution time for this model must be low in order to ensure

262 efficient operations. The second model will be executed after the coordina-
 263 tor determines when and where each aircraft would run out of fuel. This
 264 will give the coordinator, pilots, and ground crew sufficient time to make
 265 the necessary preparations. Hence, it is important to note that even though
 266 the two models are related, they work to solve different and independent
 267 situations.

268 **Problem formulation**

269 In this section, we formulate our problems as mathematical programming
 270 models.

271 **Model for allocation of aerial resources to flight routes**

272 **Notation and decision variables**

273 **Sets**

274 i, i', \mathcal{I} = indices and current set of aerial resources involved in the extin-
 275 guishing protocol.

276 g, \mathcal{G} = index and current set of aerial resources groups.

277 Each group represents all those aerial resources that can be integrated
 278 (all of them or only a proper subset) in the same flight route. We have
 279 explained the interest and the construction of flight routes in the previous
 280 section called “The problems”.

281 p, \mathcal{P} = index and current set of water recharge points.

282 k, k', \mathcal{K} = indices and current set of fire fronts.

283 \mathcal{P}_i = the current set of water points that can be assigned to resource i .

284 \mathcal{G}_i = the current group of aerial resources to which resource i is assigned.

285 **Parameters**

286 CAP_i = the carrying capacity in liters of resource i .

287 NUI_{gpk} = the maximum number of resources of group g that can be
288 assigned to the flight route given by fire front k and water recharge point p .

289 DOI_{gpk} = the number of downloads per hour performed by an aerial
290 resource of group g in the flight route, given by fire front k and water recharge
291 point p .

292 DIS_{ik} = the distance from the current position of aerial resource i to fire
293 front k .

294 NUW_p = the number of current flight routes that can share water recharge
295 point p .

296 PER_k = the percentage of water (expressed as the relative frequency)
297 intended for front k .

298 **Decision variables**

299 We use four sets of decision variables in our formulation.

300 a_{ipk} = the binary variable that takes value 1 if aerial resource i is assigned
301 to a flight route given by water point p and fire front k , and 0 otherwise.

302 m_k = the real variable that measures lack of water used in fire front k
303 relative to the amount initially assigned (sometimes this allocation may not
304 be satisfied in full).

305 f_k = the binary variable that takes value 1 if fire front k is left unattended,
306 and 0 otherwise.

307 w_{gpk} = the binary variable that takes value 1 if group g is assigned to a
308 flight route given by water point p and fire front k , and 0 otherwise.

309 Objective function

$$\begin{aligned}
\max \quad & \sum_{i \in \mathcal{I}} \sum_{g \in \mathcal{G}_i} \sum_{p \in \mathcal{P}_i} \sum_{k \in \mathcal{K}} DOI_{gpk} CAP_i a_{ipk} - \sum_{k \in \mathcal{K}} M (m_k + f_k) \\
& - \sum_{i \in \mathcal{I}} \sum_{p \in \mathcal{P}_i} \sum_{k \in \mathcal{K}} \frac{DIS_{ik}}{\max_{i' \in \mathcal{I}, k' \in \mathcal{K}} DIS_{i'k'}} a_{ipk}
\end{aligned} \tag{1}$$

310 Here, M is a constant with a sufficiently large value. We use M to give
311 priority to the minimization of the difference between actual and assigned
312 water use for all fire fronts. In addition, in the term involving distances
313 from aerial resources to fronts, a proportion is used instead of using DIS_{ik}
314 directly, so that the distances are rescaled making smaller distances even
315 smaller. Therefore, in case of ties during resource selection, the closest ones
316 will be chosen accordingly.

317 Constraints

318 The relationships that describe the real-world model are translated in our
319 formulation via mathematical constraints.

$$\sum_{p \in \mathcal{P}_i} \sum_{k \in \mathcal{K}} a_{ipk} = 1, \quad \forall i \in \mathcal{I} \tag{2}$$

$$\sum_{i \in \mathcal{I}: g \in \mathcal{G}_i, p \in \mathcal{P}_i} a_{ipk} \leq NUI_{gpk}, \quad \forall g \in \mathcal{G}, \forall p \in \mathcal{P}, \forall k \in \mathcal{K} \tag{3}$$

$$\sum_{i \in \mathcal{I}} \sum_{p \in \mathcal{P}_i} a_{ipk} \geq 1 - f_k, \quad \forall k \in \mathcal{K} \tag{4}$$

$$\sum_{g \in \mathcal{G}} \sum_{k \in \mathcal{K}} w_{gpk} \leq NUW_p, \quad \forall p \in \mathcal{P} \quad (5)$$

$$w_{gpk} \geq a_{ipk}, \quad \forall i \in \mathcal{I}, \forall g \in \mathcal{G}_i, \forall p \in \mathcal{P}_i, \forall k \in \mathcal{K} \quad (6)$$

$$\sum_{i \in \mathcal{I}} \sum_{p \in \mathcal{P}_i} CAP_i a_{ipk} \leq PER_k \sum_{i \in \mathcal{I}} CAP_i + m_k, \quad \forall k \in \mathcal{K} \quad (7)$$

320 The goal of this model is to assign a current set of aerial resources to
 321 a current set of flight routes such that the greatest possible download of
 322 water/retardant is achieved (this objective corresponds to equation (1)). In
 323 addition, we take into account the distances between air resources and fronts.
 324 Also it aims to minimize the total of such distances, but as a result of rescal-
 325 ing used in the third term, this second goal we are giving less weight.

326 A number of restrictions must be taken into account. First, each aerial
 327 resource must be assigned to a single flight route (equation (2)), and the
 328 maximum number of aerial resources in a flight route must not be exceeded
 329 (equation (3))¹ The fact that many aerial resources fly along the same route
 330 implies that there is little space between them; thus, inefficient management
 331 may result in collisions or loss of valuable time. In addition, each of the fronts
 332 of the fire must be assigned at least one aerial resource. That is, no front

¹Equation (2) specifies that each aerial resource must be assigned to a single flight route. This may lead to an infeasibility in the model when there is insufficient airspace along the flight routes for all considered aerial resources (equation (3) will not be satisfied). We assume that the air coordinator selects a number of aerial resources taking this fact into consideration. In the case of that this infeasibility occurs, a slack variable must be added that represents the violation of equation (2). This would result in a slight variation of the model, which does not pose significant difficulty, to obtain a more clear and concise representation.

333 considered by the coordinator may be left unattended (equation (4)). It is
334 important to determine which aerial resources can access the water points.
335 If the operation takes place along a coastal region, the aerial resources may
336 recharge at sea simultaneously, if conditions permit. However, if the charging
337 point is a pit, in many cases, it is accessible only by certain aerial resources,
338 such as helicopters equipped with helitanks; in such scenarios, access may be
339 impossible for an airplane. Accordingly, the number of aerial resources that
340 can recharge simultaneously at a single water point will not exceed a certain
341 number, e.g., in the case of a lake, a pit, or a water truck repurposed for fire
342 extinguishing activities. Moreover, when determining the maximum number
343 of aerial resources per flight route, the aerial resources on the same route
344 must not coincide at the same recharge point. Thus, the maximum number
345 of aerial resources that can be recharged at the same point will correspond
346 to the number of flight routes (equations (5) and (6)). The arrangement of
347 aerial resources in the various fronts must conform to the priorities assigned
348 by the aerial coordinator. In the case of Spain, this priority takes into account
349 the sum of the capacities of water and retardant that can be transported by
350 the aerial resources. Because each front is assigned a specific percentage of
351 the total amount of water resources based on its severity, an attempt is made
352 to assign each of these fronts with aerial resources whose capacity is rated
353 for that front, to a feasible extent (equation (7)).

354 **Model for allocation of aerial resources to refueling points**

355 We present the different elements that comprise the second model of opera-
356 tional research.

357 Notation and decision variables

358 Sets

359 i, \mathcal{I} = index and current set of aerial resources involved in the extin-
360 guishing protocol.

361 b, \mathcal{B} = index and current set of refueling bases.

362 t, \mathcal{T} = index and current set of periods of time (we can take each period
363 equal to five minutes, for example).

364 \mathcal{B}_i = the current set of refueling bases that can be assigned to resource
365 i .

366 It is important to note that these sets also allow the decision maker to
367 introduce heterogeneity to the aerial resources fleet because the points at
368 which each aerial resource i can refuel is determined. By using these, it
369 is made clear that not all aerial resources can refuel at the same refueling
370 points. For example, a helicopter may land in a much smaller area than a
371 fixed-wing aircraft.

372 t', \mathcal{T}_t = index and set of periods of time no later than t , *i.e.*, $\mathcal{T}_t =$
373 $\{1, \dots, t\}$.

374 Parameters

375 LOI_i = the fuel load of aerial resource i .

376 REF_i = the refueling time of aerial resource i .

377 FUE_b = the current quantity of fuel available in base b .

378 NUM_b = the number of aerial resources that can refuel simultaneously
379 on base b .

380 TIM_{ib} = the time it takes to move aerial resource i from its current

381 location to base b .

382 ATI_t = the accumulated time since the start of the refueling planning
 383 process (that is, when a set of aerial resources request refueling in the air
 384 before they actually run out of fuel) up to t period.

385 Decision variables

386 We use two sets of decision variables in our formulation.

387 s_{ibt} = the binary variable that takes the value 1 when aerial resource i
 388 starts refueling in base b in period t , and 0 otherwise.

389 e_{ibt} = the binary variable that takes the value 1 when aerial resource i
 390 ends refueling in base b in period t , and 0 otherwise.

391 Objective function

$$\min \sum_{i \in \mathcal{I}} \sum_{b \in \mathcal{B}_i} \sum_{t \in \mathcal{T}} (ATI_t + TIM_{ib}) e_{ibt} \quad (8)$$

392 Constraints

393 The relationships that describe the real-world model of the allocation of
 394 aerial resources to refueling points are formulated by means of the following
 395 constraints.

$$\sum_{i \in \mathcal{I}: b \in \mathcal{B}_i} \sum_{t' \in \mathcal{T}_t} s_{ibt'} - \sum_{i \in \mathcal{I}: b \in \mathcal{B}_i} \sum_{t' \in \mathcal{T}_t} e_{ibt'} \leq NUM_b, \quad \forall b \in \mathcal{B}, \forall t \in \mathcal{T} \quad (9)$$

$$\sum_{b \in \mathcal{B}_i} \sum_{t \in \mathcal{T}} s_{ibt} = 1, \quad \forall i \in \mathcal{I} \quad (10)$$

$$\sum_{t \in \mathcal{T}} s_{ibt} = \sum_{t \in \mathcal{T}} e_{ibt}, \quad \forall i \in \mathcal{I}, \forall b \in \mathcal{B}_i \quad (11)$$

$$\sum_{b \in \mathcal{B}_i} \sum_{t \in \mathcal{T}} (ATI_t - TIM_{ib}) s_{ibt} \geq 0, \quad \forall i \in \mathcal{I} \quad (12)$$

$$\sum_{b \in \mathcal{B}_i} \sum_{t \in \mathcal{T}} (ATI_t + REF_i) s_{ibt} - \sum_{b \in \mathcal{B}_i} \sum_{t \in \mathcal{T}} ATI_t e_{ibt} = 0, \quad \forall i \in \mathcal{I} \quad (13)$$

$$\sum_{i \in \mathcal{I}: b \in \mathcal{B}_i} \sum_{t \in \mathcal{T}} LOI_i s_{ibt} \leq FUE_b, \quad \forall b \in \mathcal{B} \quad (14)$$

396 Depending on the type of the refueling base (tanker, airfield, or airport),
 397 the number of aerial resources that can be refueled simultaneously may vary.
 398 For example, in the case of a tanker, only a hose can be provided to supply
 399 fuel to aerial resources, or serving space may be limited. To make the model
 400 consistent with this limitation, a restriction is included that takes into ac-
 401 count the availability of fuel supply to an aerial resource in a given period
 402 for a given refueling base (equation (9)).

403 With these elements, we determine the period when each aerial resource
 404 starts and ends refueling at the corresponding base (equations (10) and (11)).
 405 Refueling begins once the aerial resource has reached the base and this is
 406 available for refueling, given that the means employed for the refueling oper-
 407 ation may be preoccupied (equation (12)). On the other hand, we consider
 408 refueling to be complete when the refueling time associated with each aerial
 409 resource is accomplished. The refueling time for each aerial resource is im-
 410 portant because, depending on the type and model of aerial resources, the

411 time required may vary (equation (13)).

412 Because refueling bases often have limited fuel, the capacity to refuel
413 must be considered. Therefore, before assigning a set of aerial resources to
414 a particular base, determining fuel availability is necessary (equation (14)).

415 Given the above restrictions, we seek an allocation of aerial resources
416 at various refueling points whereby the time spent on the operation (this
417 time includes the actual refueling time as well as the flying times to and
418 from the base, so we are double counting the flying time for the air resource
419 to the refueling base) is minimized (equation (8)). It is worth noting that
420 in the objective function the parameter ATI_t is multiplied by the indicator
421 variable e_{ibt} . It is thus possible that different air resources begin to refuel in
422 different bases at different times. By taking all these elements into account,
423 the efficiency of the refueling operation as well as the management of fuel
424 stocks can be improved, thus avoiding supply problems.

425 **Examples, numerical results, and sensitivity analysis**

426 The above models were programmed using **AMPL** (Fourer *et al.*, 1993) and
427 solved using the Gurobi solver (Gurobi Optimization, 2016). We work with
428 four databases containing information on the wildfire fronts , the water
429 points, the available aerial resources, and the refueling bases. Although,
430 according to the title of the work, our models would be applicable to an
431 extinguishing operation that makes use of both helicopters and airplanes, in
432 the examples and with the intention of simplifying the presentation, we only
433 considered helicopters. In a more general case, the treatment of one type

434 or another would be different; this is reflected by the fact that the type of
435 aerial resources will determine the permissible type of flight route and the
436 permissible type of refueling base selected.

437 **Assignment of aerial resources to flight routes**

438 Prior to the execution of the program, information about the maximum
439 number of aerial resources per flight route and the discharges per hour of
440 every aerial resources is known.

441 The first model is demonstrated by an example (the data for models of
442 the examples are real, and were obtained from the websites of the helicopters
443 that are used. The data, described in the Appendix (Tables A1-A8), are
444 inspired by a real situation. The availability of two groups of aerial resources
445 is assumed: light helicopters BellB412-1, BellB412-2, BellB212-1, BellB212-
446 2, BellB407-1, BellB407-2, and BellB407-3 (which can enter the same flight
447 route) and heavy helicopters Ka32-1, Ka32-2, and Ka32-3 (which also can
448 enter the same flight route). Further, it is assumed that there are three
449 wildfire fronts, and from each front, ten water loading points can be accessed.
450 Three helicopters can use all the water points, another three helicopters can
451 use four water points, and the remaining four helicopters can use seven water
452 points. In this case, the model has 6 continuous variables and 273 binary
453 variables. Table 1 summarizes the main results concerning the assignment
454 of aerial resources to fronts and water points.

Table 1: Allocation of aerial resources to flight route (fire front and water point).

Aerial resources/Flight route	F1-P3	F1-P6	F2-P3	F2-P7	F3-P1	F3-P6
BellB412-1 (group 1)	0	0	1	0	0	0
BellB412-2 (group 1)	0	0	0	0	1	0
BellB212-1 (group 1)	0	0	0	1	0	0
BellB212-2 (group 1)	0	0	0	1	0	0
BellB407-1 (group 1)	0	1	0	0	0	0
BellB407-2 (group 1)	0	0	0	0	0	1
BellB407-3 (group 1)	0	1	0	0	0	0
Ka32-1 (group 2)	1	0	0	0	0	0
Ka32-2 (group 2)	1	0	0	0	0	0
Ka32-3 (group 2)	0	0	0	0	1	0

455 In Table 1, 1 indicates that the aerial resource (in rows) is assigned to
456 the flight route that can be formed by joining corresponding front and water
457 point (in columns). Otherwise, the number is assigned is 0. We note that 10
458 aerial resources are used. In addition, seven flight routes are formed. Two of
459 the flight routes traverse front 1 and use water points 3 and 6, respectively.
460 One of the flight routes is formed by two helicopters in group 2 (Ka32-
461 1 and Ka32-2), and the other one is formed by two helicopters in group
462 1 (BellB407-1 and BellB407-3). One flight route traverses front 2; it uses
463 water point 7, and it is formed by the two helicopters in group 1 (BellB212-1
464 and BellB212-2). The remaining flight routes are organized using just one
465 helicopter.

466 Moreover, the consistency of the results should be noted. For example,
467 two of the helicopters with the most water capacity are assigned to front 1
468 and they are on the same flight route; however, these helicopters are not the
469 closest to the front. Now, along this flight route, these types of helicopters
470 are very efficient. Front 1 requires the most water because it represents
471 a greater threat. Front 1 also has another flight route, used in this case
472 by two of the lightest helicopters. The other heavy helicopter, the other
473 light helicopter, and one intermediate helicopter are assigned to the front
474 3, which has the second highest water requirement. Finally, the remaining
475 three medium capacity helicopters are assigned to front 2, which requires
476 the least amount of water. From the results, it can be seen that 44.51 % of
477 capacity is expended at front 1, 25.08 % at front 2, and 30.41 % at front 3,
478 which is very close to the percentages specified in the database corresponding
479 to the fronts, which are 45 %, 25 % and 30 %, respectively.

480 To perform a sensitivity analysis, while maintaining the characteristics
481 of the fire, the effect of modifying the fleet of helicopters on the allocation is
482 analyzed. We explain some of the results obtained below. When two of the
483 large helicopters are not considered, then two medium helicopter and one
484 small helicopter take over for front 1, while the remaining large helicopter
485 is assigned to front 3, just as in the initial example. However, as regarding
486 the initial example, when two small helicopters are not considered, we see
487 that each of the big helicopters is assigned to a different front. Finally, when
488 two medium helicopters and one small helicopter are not considered, then
489 two large helicopters are assigned to front 1, while front 3 receives one large
490 helicopter and one small helicopter; two medium and one small helicopter
491 are assigned to front 2. In this case, the percentage of water allocated to the
492 fronts is 45.36%, 26.49%, and 28.15%, respectively.

493 We have also analyzed the effect of modifying the groups. For example,
494 we have replaced three air resources in group 1 with another three in group 2,
495 which are resources with more capacity. The result has been that the amount
496 of water discharged has been increased and the allocations of resources to
497 the flight routes have changed, while the rest of the results have remained
498 similar. We have also been interested in finding out the effect of modifying
499 distances from the current position of air resources to fire or to water points.
500 For example, starting from the initial case, one of the resources with the
501 most capacity has been considered in a current position closer to the fire.
502 The effect has been a change in the allocation of resources to flight routes, a
503 decrease in the total distance and the remaining results remained unchanged.

504 **Allocation of aerial resources to refueling points**

505 To demonstrate the use of the second model by an example, we assumed
506 a single scenario with four helicopters and three refueling points. Two heli-
507 copters can access up to two bases, one helicopter has access to one base, and
508 one helicopter has access to all the bases. It is assumed that the helicopters
509 have to rest. The data corresponding to the given scenario can be seen in
510 Appendix (Tables A9-A13).

511 In Table 2, 1 indicates that the corresponding helicopter is assigned to
512 the corresponding base. It must be noted that two helicopters are assigned
513 to two bases (each helicopter to a base) bases 1 and 2, and two helicopters
514 are assigned to base 3. The latter accommodates the largest helicopter and
515 one of the lightest, and has the most fuel available and the only base that can
516 serve the large helicopter. Moreover, other scenarios are possible wherein all
517 the bases are not used. However, it is not possible to carry out the entire
518 refueling operation in the least amount of time without using all the three
519 bases. On the other hand, the results show that at each base, there is, at
520 most one helicopter in each time period and that the helicopters begin to
521 refuel in different periods.

Table 2: Allocation of aerial resources to bases (refueling points).

	BellB412	BellB212	Ka32	BellB407
B1	0	0	0	1
B2	1	0	0	0
B3	0	1	1	0

522 The amount of fuel remaining at each base after this first round of refu-
523 eling is 300, 450, and 2136 liters, respectively. From the results, we can see
524 that all the bases decrease their initial capacity by more than 50 % but less
525 than 75 %. Moreover, we can see that only one aerial resource is forced to
526 wait for the other helicopter to finish refueling.

527 The optimal value of the objective function is 120 minutes. This value
528 represents the sum of the refueling time of the helicopters, including a round
529 trip to the base and the waiting time. In other words, the plan assigns each
530 helicopter to the closest possible base, and meets the condition for sufficient
531 capacity. As with the first model, a post-optimality analysis was performed
532 with new experiments to illustrate the consistent sensitivity in the model. To
533 cite an example, by eliminating the heaviest helicopter, the allocation for the
534 other three helicopters remains the same as in the initial example, when the
535 helicopters were assigned to the nearest bases. In the new scenario, there are
536 no waiting times and the total time decreases to 80 minutes. If instead of the
537 heaviest one, we eliminate the lighter helicopter, the time decreases to 77.5
538 minutes. This may be attributed to the fact that the heaviest helicopter,
539 which requires more time to refuel, is closer to the assigned base. This
540 compensation results in a reduction in the total time. Finally, regarding the
541 initial case, we consider an additional small air resource, close to base 1 and
542 suppose also that the available quantities of fuel increase in bases 1 and 3.
543 The result is that the allocation to the bases of the initial resources does not
544 changes and the additional resource is allocated to base 1. With this, the
545 total time of the operation increases.

Table 3: Executing times (in seconds) with Gurobi: Model 1.

Aerial Resources	Water Points	Fire Fronts	Mean Time	S.D. Time	Max. Time
3	3	3	0.0380	0.1274	0.91
3	6	3	0.0265	0.0715	0.48
3	6	6	0.2456	0.3824	1.66
3	9	3	0.0289	0.0604	0.37
3	9	6	0.2622	0.6218	3.28
6	3	3	0.3404	0.4195	1.87
6	6	3	0.3477	0.4006	1.39
6	6	6	1.2028	1.6589	8.18
6	9	3	0.3051	0.3542	1.59
6	9	6	0.8273	0.9696	4.40
9	3	3	0.8799	0.9659	5.65
9	6	3	2.4459	6.2381	44.11
9	6	6	14.5958	25.2978	145.69
9	9	3	0.6675	0.5205	2.20
9	9	6	9.0187	15.8180	89.63
12	3	3	7.0417	18.9782	137.49
12	6	3	13.5069	25.6252	158.60
12	6	6	7.0090	9.3167	44.88
12	9	3	8.1102	15.3645	83.23
12	9	6	13.2312	23.3966	137.64
15	3	3	28.9217	90.0668	795.07
15	6	3	128.4341	713.8449	7072.86
15	6	6	19.7705	37.4855	215.98
15	9	3	30.4158	81.4541	649.52
15	9	6	55.4892	204.7346	1594.88

546 Algorithms for larger incidents

547 In this section, we explore the feasibility of solving real-size instances in
548 a reasonable execution time. We apply the two algorithms over instances
549 ranging from 3 to 15 aerial resources, from 3 to 9 water points, from 3 to 6
550 fire fronts, and from 3 to 15 refueling bases. The model algorithms can be
551 efficiently computed on a modern PC.

552 Table 3 lists the execution times (in seconds) for solving the first AMPL
553 model by using the Gurobi solver for different instances. The first column
554 indicates the number of aerial resources, the second column indicates the
555 number of water refill points, and the third column indicates the number
556 of fronts. Column 4 indicates the mean time and column 5 indicates the
557 standard deviation. Finally, column 6 shows the maximum value obtained
558 for the time of execution. The parameters for the different cases are randomly
559 generated, and for each case, we consider 100 samples.

560 It can be seen from Table 3 that the average execution time of all cases for
561 Model 1 barely exceeds two minutes for the worst case. This is illustrated in
562 Figure 1, wherein the execution times (in seconds) are plotted on the vertical
563 axis and the number of aerial resources are plotted on the horizontal axis;
564 for different types of lines, different scenarios are represented by the number
565 of fronts and water points. Clearly, the worst computational result (more
566 than 120 seconds) is obtained for 15 aerial resources, 6 water points, and 3
567 fronts. In other cases, we obtain an average computation time of less than
568 60 seconds.

Figure 1: Execution time (in seconds) with different scenarios: Model 1.

569 For higher values of the problem parameters that define the scenario
570 (number of aerial resources equal to or greater than 9), the standard deviation of runtime ranges from half a minute and reaches almost twelve minutes.
571
572 In only one case, the runtime reached nearly two hours (with 15 aerial resources, 6 water points, and 3 fire fronts).
573

574 In a similar manner, Table 4 lists the execution times (in seconds) for solving the second **AMPL** model by using the Gurobi solver for different instances.
575
576

577 As shown in Table 4, the average execution time of all cases for Model 2 barely exceeds twelve minutes in the worst case. This is illustrated in
578 Figure 2, wherein the execution times (in seconds) are plotted on the vertical axis and the number of aerial resources are plotted on the horizontal axis;
579 for different types of lines, different scenarios are represented by different number of refueling bases. Clearly, the worst computational result (more
580 than 756 seconds) is obtained for 12 aerial resources and 3 refueling bases.
581
582 In other cases, we obtain a computing average of less than 180 seconds.
583
584

585 For a large number of problem parameters that define the scenario (number of aerial resources equal to 12), the standard deviation of runtime ranges
586 between two minutes and more than thirty minutes. In only one case the runtime standard deviation reached a half hour (with 12 aerial resources and
587
588 3 refueling bases).
589

Table 4: Executing times (in seconds) with Gurobi: Model 2.

Aerial Resources	Refueling Bases	Mean Time	S.D. Time	Max. Time
3	3	0.2920	0.4497	3.9554
3	6	1.0027	3.0879	18.8991
3	9	0.9979	3.5243	31.3052
3	12	1.2722	5.6377	57.0593
3	15	1.1700	3.1200	32.0351
6	3	22.8064	31.0903	103.2493
6	6	21.5974	36.2055	185.7048
6	9	13.2956	28.0623	147.1876
6	12	7.0087	21.9127	142.3974
6	15	12.0798	49.4308	433.9180
9	3	114.0099	198.3733	1474.9258
9	6	63.1443	69.2111	262.9560
9	9	66.4848	99.1811	505.6261
9	12	74.4264	106.4668	392.9953
9	15	48.9573	110.3735	628.9864
12	3	756.9109	1948.0024	12513.7766
12	6	168.1308	179.1913	1097.7551
12	9	179.4069	147.7079	508.8336
12	12	150.0898	186.3166	867.2902
12	15	129.5944	175.3732	663.8251

Figure 2: Execution time (in seconds) with different scenarios: Model 2.

590 Conclusions and final remarks

591 In this work, we present two integer linear programming models that solve
592 two real problems experienced by traffic control coordinators when tackling
593 wildfires. In several examples inspired by real situations, we see that the
594 model resolution is fast. These features indicate, in our opinion, that this
595 work can assist air traffic control coordinators in decision making for this
596 type of fire. In fact, by using the tools proposed in this work, such as col-
597 lision avoidance algorithms, escape route design for ground crews, efficiency
598 measures of water discharges, and control of the spread of fire, this work
599 can be integrated into a more complex, holistic, and user-friendly system
600 for decision support, which also incorporates the modern methods of image
601 processing and presentation.

602 Although our models have been tested using real parameters and data,
603 the results could not be verified using historical data. It is difficult to obtain
604 specific results for resource allocation, although it is easier to obtain global
605 data. We emphasize the recommendation that aviation agencies consider the
606 importance of data and its availability.

607 The proposed models perform resource allocation involving flight routes
608 and refueling points. Therefore, it is important to note that once this as-
609 signment is made according to certain objectives and restrictions, aerial re-
610 sources must effectively integrate the task of fire extinguishing; temporal
611 assignments that include pilot work schedules, rest periods, and flight times
612 must be taken into consideration.

613 Coordinating aerial resources is a challenging task. By monitoring the ef-

614 fectiveness of our approach and making adjustments to maximize the impact
615 on the fronts, the proposed automated approach can benefit aerial coordina-
616 tors. Given the fronts, available aerial resources, water points, and refueling
617 points in a scenario, aerial resources are allocated to flight routes and refuel-
618 ing points such that the water discharge and refueling times are optimized.
619 The initial planning must be supplemented with the algorithm mentioned
620 above, which is executed based on the pilot work schedule, rest periods, and
621 active fronts. We emphasize that this last model is flexible in the sense that
622 its goals can be modified according to interest and optimal water discharge
623 or other cost/benefit functions. Variation in one of the model parameters can
624 signal the need for a rerun of any of the three algorithms and reallocations.

625 Other approaches, such as a single model for the problem presented herein
626 or a stochastic approach may make sense. We do not endorse the single model
627 because of the associated computational costs and the natural separation of
628 problems. With respect to stochastic optimization, the consideration might
629 be interesting for the allocation of aerial resources to a flight routes model
630 because the efficiency of aerial resources during a wildfire must be estimated
631 (depending on the climatological factors and the drought situation of the
632 land, among others), and thus, an open problem would be to study the allo-
633 cation of air resources to flight routes by means of a stochastic programming
634 model. However, we do not consider the introduction of stochasticity in the
635 second model, because in this case the parameters are deterministic.

636 It is important to mention that each region has a specific means to fight
637 fires; for example, in Galicia (a region in North-west Spain, with a surface
638 area of 29574 km² and a 69% mountain range), the regional public body

639 responsible for fighting forest fires lists 30 aerial resources in 2017, of which
640 25 are helicopters. These helicopters not only allow for intervention from
641 the air, but also for the transport of land brigades. They are distributed in
642 21 operational bases spread across the four provinces of Galicia. The rest
643 of the fleet comprise large seaplanes sent by the Central Administration as
644 needed, and they are able to refill from pools or swamps without needing to
645 return to their base.

646 The models developed in this work could also serve as a starting point
647 for subsequent case studies. Finally, as our first model is currently a multi-
648 objective model with emphasis on water download, creating a Pareto frontier
649 to demonstrate the trade-off between flight distance and water download may
650 be interesting.

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706 **Appendix**

707 **A Information of the examples**

708 This appendix describes the data used in examples above. Such data are
709 provided as used to run the model programmed with **AMPL**.

Table A1: Helicopter type.

Light	BellB412	BellB212	BellB407
Heavy	Ka32		

Table A2: Water points useful for helicopter types.

Helicopter type	Water points									
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
BellB412	✓	✓	✓	✓	✓		✓			✓
BellB212	✓	✓	✓	✓	✓		✓			✓
BellB407	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Ka32	✓	✓	✓		✓					

Table A3: Helicopter type capacities.

Helicopter type	BellB412	BellB212	BellB407	Ka32
Capacity (liters)	2274	2360	1205	5000

Table A4: Number of helicopters per flight route.

Light helicopters	Water points									
Front number	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
K1	2	3	2	6	3	2	2	8	3	3
K2	2	4	2	10	9	2	2	6	3	6
K3	2	4	2	10	3	2	2	4	2	5

Heavy helicopters	Water points									
Front number	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
K1	2	2	2	4	2	2	2	6	2	2
K2	2	3	2	7	7	2	2	4	2	4
K3	2	3	2	7	2	2	2	3	2	4

Table A5: Water downloads per hour depending flight route.

Light helicopters	Water points									
Fronts	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
K1	15	9	15	5	10	22	18	3	8	9
K2	18	7	22	1	3	15	24	5	9	5
K3	18	6	16	3	10	27	18	7	12	5

Heavy helicopters	Water points									
Fronts	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
K1	16	10	16	5	11	24	19	3	8	10
K2	19	7	24	1	3	16	26	5	10	5
K3	19	6	17	3	11	30	19	7	13	5

Table A6: Distance from helicopters to the fronts (hours).

Helicopters	Fronts		
	K1	K2	K3
BellB412-1	0.31	0.31	0.34
BellB412-2	0.35	0.33	0.32
BellB212-1	0.23	0.20	0.20
BellB212-2	0.21	0.22	0.24
BellB407-1	0.75	0.75	0.71
BellB407-2	0.98	0.98	0.82
BellB407-3	0.63	0.62	0.70
Ka32-1	1.36	1.55	1.54
Ka32-2	0.57	0.56	0.57
Ka32-3	0.14	0.14	0.13

Table A7: Number of helicopters charging water in same water point.

Water points	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
Num. of helicopters	5	5	5	3	3	4	5	2	2	4

Table A8: Desired percentage of water in each front.

Fronts	K1	K2	K3
Percentage	45	25	30

Table A9: Refueling points useful for helicopter types.

Helicopter type	Refueling points		
	B1	B2	B3
BellB412		✓	✓
BellB212		✓	✓
BellB407	✓	✓	✓
Ka32			✓

Table A10: Helicopters information.

Helicopters	Fuel charge (liters)	Refueling time (minutes)
BellB412	1050	7.5
BellB212	614	5.0
BellB407	400	2.5
Ka32	2250	12.5

Table A11: Bases information.

Bases	Fuel available (liters)	Maximum number of helicopters refueling simultaneously
B1	700	1
B2	1500	1
B3	5000	1

Table A12: Flight time of helicopters to the bases (minutes).

Helicopters	Bases		
	B1	B2	B3
BellB412	5.0	5.0	15.0
BellB212	25.0	25.0	15.0
Ka32	17.5	12.5	10.0
BellB407	12.5	15.0	15.0

Table A13: Time allocated to each time period.

Time periods	1	2	3	4	5	6	7	8	9	10	11	12	13
Time (minutes)	0	2.5	5	7.5	10	12.5	15	17.5	20	22.5	25	27.5	30